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SERVICE-ADAPTIVE MULTI-TYPE REPAIRMAN PROBLEMS

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Abstract. The classical "repairman problem," cf. Feller (1967) is generalized to consider r failure-prone machine types, each type having its own individual failure rate and also repair rate. Each failed machine joins its type queue, and is repaired by a single server. Several dynamic service priority schemes are considered that approximate first-come first-served, longest-line first, and least-available first. A heavy-traffic asymptotic analysis determines approximations to the time-dependent mean and covariance of individual type queue lengths, and shows that the marginal joint distribution of queue lengths is approximately Ornstein-Uhlenbeck. Numerical illustrations of approximation accuracy are provided, as are suggested applications to computer performance and manufacturing systems analysis.

1. INTRODUCTION

The classical "repairman problem" described by Feller (1967), Cox and Smith (1961), and in all later texts on applied probability, is a well-motivated example of a finite Markov chain in continuous time. Not only is it a plausible first model for the productivity of a group of similar machines that are subject to failure and susceptible to repair by one (or more) repairmen, as described by Feller and subsequent authors, but it provides initial rough models for prototypical computer science situations such as multiprocessing and polling. The purpose of this paper is to extend the basic simple *single-type* (of machine, for instance) Markov repairman model to various *multi-type* models, i.e., ones in which several types of items (e.g., machines or computer system users) become applicants for service. These then *may* join a single queue and be serviced in arrival order: first-in, first-out or FIFO. Alternatively, the service applicant types can be individually queued, and a service assignment can be made from one of these queues at a suitable instant, such as at the end of a service; evaluation of the latter *dynamic complete-service assignment* rules are emphasized in this paper. Note parenthetically that in principle there need be no restriction to choice from among the waiting types *only* when service is completed: preemption at some intermediate time can be an effective tactic, particularly if setup or switchover times or costs are not excessive, in order to focus attention on a machine type that momentarily has a long queue and thus is in excessively short supply. One limit of preemption is some form of processor sharing, in which the server's tendency to service a particular type among service applicants depends moment by moment upon lengths of queues, or urgency. In some contexts switching back and forth between partially completed services is severely penalized by substantial switchover times, which reduces the attractiveness of the tactic. It is even possible that such switching can lead to bistability, an elementary form of chaos; see Jaiswal, *et al.* (1990). We do not consider such switching models in the present paper.

In what follows we first describe our mathematical model, paying particular attention to a handy probabilistic device for representing purposeful and adaptive selection of a new service incumbent just after the previous one completes service and leaves. An important example could be the simple practice of selecting from the longest queue; another is selecting from the queue of items that is least available. We then introduce a state space scaling that is appropriate when the number of each machine type grows large and the system is in heavy traffic, so the server is essentially never idle. Asymptotic expansions are introduced in terms of a large parameter, a; a is of the order of magnitude of the number of machines. These expansions reveal the approximate time-dependent means of the lengths of the queues awaiting processing, and also the time-dependent joint characteristic function (ch.f.) of the random noise terms that characterize departures from the mean. The latter approaches the ch.f. of a multivariate Ornstein-Uhlenbeck process; see Arnold (1974). The scaling studied is the same as that of McNeil and Schach (1973) but the present results pertain specifically to a multi-type queueing system's transient behavior. Finally, numerical results computed from the model by numerical solutions of systems of differential equations are compared to simulations, with good agreement exhibited. The computational time for the presently described analytical-numerical procedure is a small fraction of that required to conduct a comparable simple simulation, suggesting its practicality and usefulness when appropriate.

2. THE MODEL

In this section we describe the initial basic model for a multi-type repairman problem with adaptive service.

a) Demands

There are K_j items ("machines") of type j, each of which, when operating, fail independently at Markovian rate λ_j; $j = 1, 2, ..., r$. In fact, the failure rate can even be smoothly time-dependent, with failure rate at time (*not* age) t equal to $\lambda_j(t)$, but we emphasize discussion of the time-homogeneous situation here.

b) Service

There is a single server. When an item of type j is undergoing service its repair process is Markovian with rate υ_j. Again, time-dependence is allowable. We only consider the case of *heavy traffic*, which means that $\sum_{j=1}^{r} K_j\lambda_j / \upsilon_j > 1$ when rates are constant. This assumption assures that queues for all items form soon after the process begins; the queues grow towards steady-state levels and fluctuate but essentially never dissipate.

c) Service Discipline; Models for Adaptation

Service is organized by assembling different demand types in their own queues. Let $N_j(t)$ denote the number of items of type j awaiting service at time t; if a type j item happens to be undergoing service it is also included in the count for $N_j(t)$. Let $I(t)$ be an indicator variable; $I(t) = i$ ($i = 1, 2, ..., r$) means that an item of type i is in service at time t.

When an item of any type completes service its replacement is *modeled* as if selected by a random device: given that service is completed at time t and the number of items in the various queues is $\underline{N}(t) = \underline{n}$ then with probability $q_j(\underline{n}, \underline{K}, \underline{w}(t))$ the new server incumbent at t is of type j. The function $q_j(\cdot)$ is to be chosen so as to model desired service protocols; it is, in particular, assumed to be homogeneous of degree zero in $\underline{n}$ and $\underline{K}$. We select for study specific forms that represent, for example, (i) first-in, first-out (FIFO) service selection, (ii) longest-line-next (LOLIN), (iii) least-available-next (LAIN) or any weighted combination thereof. Here are the explicit forms chosen; for $j = 1, 2, \ldots, r$

(*i*) FIFO:
$$q_j(\underline{n}(t), \underline{K}, \underline{w}(t)) = \frac{n_j(t) w_j(t)}{\sum_k n_k(t) w_k(t)} \tag{2.1}$$

(*ii*) LOLIN:
$$q_j(\underline{n}(t), \underline{K}, \underline{w}(t)) = \frac{\left(n_j(t)\right)^p w_j(t)}{\sum_k \left(n_k(t)\right)^p w_k(t)}, p \gg 1 \tag{2.2}$$

(*iii*) LAIN:
$$q_j(\underline{n}(t), \underline{K}, \underline{w}(t)) = \frac{\left(K_j - n_j(t)\right)^{-p} w_j(t)}{\sum_k \left(K_k - n_k(t)\right)^{-p} w_k(t)}, p \gg 1 \tag{2.3}$$

where $(w_j(t)) = \underline{w}(t)$ is a set of positive weights and p is a positive tuning constant (suitably chosen to be "large, but not too large"). Note that each specific function is the ratio of homogeneous functions, differentiable in n_j and also K_j. To motivate (2.3), it is clear that if, say, the availability of item 1, namely $K_1 - n_1$ is less than that of any other item $j \neq 1$, then its *anti-availability* $(K_1 - n_1)^{-1}$ is greater than that of any other, and the difference is enhanced by increasing p in $(K_l - n_l)^{-p}$; consequently the q_j of (2.3) increases controllably the probability of choosing item j for next service if it is least available. In principle ambiguity exists if $n_k(t) = K_k$ for more than one machine; we do not consider this unlikely event. An analogous argument holds for (2.2). It has long been understood that (2.1) with equal weights already well-approximates FIFO; its use eliminates the need for a far more elaborate state space to describe true FIFO. See Gaver and Morrison (1991) for a treatment of our FIFO model by somewhat different methods.

Although we model the selection of the next item to be served by a random process governed by q_j the intention is that this model emulate deterministic selection. Modeling by a probability that is a sufficiently smooth (differentiable) function of state variables $n_j(t)$ permits computation of asymptotic expansions that conveniently characterize measures of the probabilistic fluctuations of those state variables; in particular the approximate variance-covariance function of $\underline{N}(t)$. It will be shown by simulation for specific examples that numerical agreement of our probabilistic analytical models is very close to the corresponding deterministic selection results. The computing time for obtaining the numerical results from the analytical model is, however, a small fraction of the time to obtain the simulation results. This allows considerable elaboration of the present basic model; for some details see Gaver *et al.* (1991).

There follows a discussion of application areas for our class of models.

Adaptive Polling

Suppose a single processor, e.g., computer or database, receives tasks from r different classes of input devices which we will call terminals. When a terminal in class j makes a demand it remains quiescent until that demand is served, after which it becomes active again at rate λ_j. The requests from each terminal individually queue for service at the processor; when a request or task is accepted by the (single) processor it is served to completion; the server is said to **poll** terminal class queues for the next request or task to be served when completion occurs. The analysis herein permits assessment of various polling selection policies, such as one that dynamically favors the longest queue (equally-weighted LOLIN) and thus tends to equalize queue lengths and waits for service. If the terminal classes were to be information sources in a military command-control-intelligence (C^2I) system, and requests were database updates, the priority weights, w_j, could be adjusted to account for the current credibility of inputs by classes of terminals; the most credible would be processed first in times of stress and heavy usage, as during combat. It is easily possible to adjust the service policy to that of class-preferential processor sharing, see Gaver and Jacobs (1986) and Morrison (1987) for similar asymptotic studies.

Why LAIN? Promotion of Item-Cooperative Service in Reliability or Manufacturing Applications

Several real situations call for the use of the least-available-next (LAIN) policy as at least attractive, if possibly not demonstrably optimal. Consider the modeling of adaptive service to achieve

System Availability

First, suppose that exactly one of each item type is needed in order to render fully operational a *platform system* such as a fully-equipped aircraft. Then the least-available item type controls the number of platforms available for action. In case there are fewer of each item type than there are platforms then all available platforms will be in action at any time and the simple process described above is in play and its analysis is appropriate. For a more detailed description of the aircraft availability process see Gaver *et al.* (1991).

Manufacturing: Stage-Wise Production

Second, suppose instead that each item type represents a machine that is a stage in some sequential manufacturing process. Manufacturing, or other processing, proceeds in successive stages, starting at a bank of (available) machines of type 1, the output of that stage proceeding to, and through, machines of type 2, *etc.*, through machines of type r at Stage r. Machines are, however, susceptible to failure and only those that have not failed (are available) are productive. Provided that all machine banks are about equally productive, so that there are no production bottlenecks in the absence of failures, it is of interest to schedule repairs to maintain that balance when realistic failures do occur. Certainly the LAIN policy has the desired effect of maintaining equality among machine types available. A generalization might well be to induct for next service that machine type that has the largest queue of in-process production items awaiting that particular machine bank.

3. MARKOV STRUCTURE AND ASYMPTOTIC ANALYSIS

It is clear from the assumptions made that the process $\{\underline{N}(t), I(t)\} = \{N_1(t), N_2(t), \ldots N_r(t), I(t)\}$ is Markov in continuous time with a finite discrete state space. If all parameters are actually independent of time such a process will ultimately reach steady-state, exhibiting a stationary distribution. But explicit calculation of that distribution, or of the distribution at any finite time, t, is an involved exercise. Special efforts would be required for system design and operation. The same is true if simulation is used to study the behavior of various scheduling options such as FIFO, LOLIN, or LAIN. Hence the approach of approximating the behavior of the above multivariate process by the sum of a deterministic mean and a continuous-state-space diffusion process with Gaussian marginal distributions is attractive. McNeil and Schach (1973) carried out a comparable program for a wide variety of models, but our approach and results differ from theirs in various respects; in particular our process is multivariate.

Asymptotic Analysis

Let

$$P_\ell(\underline{n},t) = P\{N_1(t) = n_1, \ldots N_r(t) = n_r, I(t) = \ell\} \equiv P\{\underline{N}(t) = \underline{n}, I(t) = \ell\}, \quad (3.1)$$

where initial conditions have been suppressed. Then standard arguments lead to the forward Kolmogorov equations

$$\frac{\partial}{\partial t}P_\ell(\underline{n},t) + \sum_{j=1}^{r}\lambda_j(K_j - n_j)P_\ell(\underline{n},t) + \upsilon_\ell P_\ell(\underline{n},t) =$$

$$\sum_{j=1}^{r}\lambda_j(K_j - n_j + 1)P_\ell(\underline{n} - \underline{e}_j,t) + q_\ell(\underline{n},\underline{K})\sum_{j=1}^{r}\upsilon_j P_j(\underline{n} + \underline{e}_j,t), \quad (3.2)$$

for $n_\ell \neq 0$, $\underline{n} \neq \underline{e}_\ell$ and $\underline{0} \leq \underline{n} \leq \underline{k}$, where $\underline{e}_j = (0, 0, ..., 1, 0, 0)$, i.e., 1 occurs only in the j^{th} entry of vector $\underline{e}_j$. We do not include the special equations appropriate when $\underline{n} = \underline{0}$ and $\underline{n} = \underline{e}_\ell$ since the heavy traffic assumption renders this unnecessary.

Scaling

For a large value a, i.e., a»1 we assume $K_j = a\alpha_j$, $v_j = a\mu_j$. Consider the transformed variables

$$V_j(t) = \left(N_j(t) - a\beta_j(t)\right) / \sqrt{a}, j = 1,2,\ldots,r \tag{3.3}$$

wherein $\beta_j(t)$ are deterministic functions to be determined; $a\beta_j(t) \simeq E[N_j(t)]$. Define the characteristic function (ch.f.) of $V_j(t)$ for given a as

$$\begin{aligned}\omega_\ell(y,t;a) &= E\left[\exp\left(i\underline{y}\underline{V}(t)\right); I(t) = \ell\right] \\ &= \sum_{(\underline{n})} \exp\left[i\underline{y}\left(\underline{n} - a\underline{\beta}(t)\right) / \sqrt{a}\right] P_\ell(\underline{n},t).\end{aligned} \tag{3.4}$$

The objective will be to derive asymptotic expansions in powers of $1/\sqrt{a}$ that characterize the mean function $\underline{\beta}(t) = (\beta_1(t), \beta_2(t), ..., \beta_r(t))$ and also $\underline{\omega} = (\omega_1, \omega_2, ..., \omega_r)$; from the latter variances and covariances of queue lengths can be deduced. From the limiting characteristic function it may be deduced that the process $\underline{V}(t)$ is approximately multivariate Ornstein-Uhlenbeck, with parameters specific to the particular scheduling rule in force.

Preliminaries

According to its definition, (3.4),

$$\omega_\ell(y,t;a) = \sum_{(\underline{n})} e^{i\underline{y}\left(\underline{n} - a\underline{\beta}(t)\right)/\sqrt{a}} P_\ell(\underline{n},t) \equiv \sum_{(\underline{n})} e^{i\underline{y}\underline{v}(\underline{n},t)} P_\ell(\underline{n},t) \tag{3.5}$$

where

$$\underline{v}(\underline{n},t) = \left(\underline{n} - a\underline{\beta}(t)\right) / \sqrt{a}. \tag{3.6}$$

Now multiply (3.2) by $e^{i\underline{y}\underline{v}}$ and sum on $\underline{n}$. Note that differentiation of (3.5) gives

$$\frac{\partial \omega_\ell}{\partial t} = -i\sqrt{a}\,\underline{y}\frac{d\underline{\beta}}{dt}\omega_\ell + \sum_{(\underline{n})} e^{i\underline{y}\underline{v}} \frac{\partial}{\partial t} P_\ell(\underline{n},t), \tag{3.7}$$

and that the scaling (3.6) implies

$$\begin{aligned}\sum_{(\underline{n})} e^{i\underline{y}\underline{v}} \sum_{j=1}^{r} n_j P_\ell(\underline{n},t) &= \sum_{j=1}^{r}\sum_{(\underline{n})}\left(\sqrt{a}v_j + a\beta_j(t)\right)e^{i\underline{y}\underline{v}} P_\ell(\underline{n},t) \\ &= \sum_{j=1}^{r}\left(-i\sqrt{a}\frac{\partial \omega_\ell}{\partial y_j} + a\beta_j(t)\omega_\ell\right),\end{aligned} \tag{3.8}$$

so from the Kolmogorov equation

$$\frac{\partial \omega_\ell}{\partial t} + i\sqrt{a}\,\underline{y}\frac{d\underline{\beta}}{dt}\omega_\ell + \sum_{j=1}^{r}\lambda_j\left[K_j\omega_\ell + i\sqrt{a}\frac{\partial \omega_\ell}{\partial y_j} - a\beta_j(t)\omega_\ell\right] + a\mu_\ell\omega_\ell =$$

$$\sum_{j=1}^{r}\lambda_j\left[K_j\omega_\ell + i\sqrt{a}\frac{\partial \omega_\ell}{\partial y_j} - a\beta_j\omega_\ell\right]e^{iy_j/\sqrt{a}} + \sum_{(\underline{n})} e^{i\underline{y}\underline{v}} q_\ell(\underline{n},\underline{K},\underline{w})\sum_{j=1}^{r} v_j P_j\left(\underline{n}+\underline{e}_j,t\right). \tag{3.9}$$

Expand q_j by noting that in representation (2.1)–(2.3) numerator and denominator are homogeneous functions, so cancellation of $\sqrt{a}$ provides

$$q_j\left(\underline{n},\underline{K},\underline{w}(t)\right) = q_j\left(\underline{\beta}(t),\underline{\alpha},\underline{w}(t)\right) + \frac{1}{\sqrt{a}}\sum_{k=1}^{r} v_k\frac{\partial q_j}{\partial \beta_k} + O(1/a). \tag{3.10}$$

Now insert (3.10) into the final sum of (3.9) to get

$$\frac{\partial \omega_\ell}{\partial t} + i\sqrt{a}\,\omega_\ell \underline{y}\frac{d\beta}{dt} + \sum_{j=1}^{r}\lambda_j\left[a\alpha_j\omega_\ell + i\sqrt{a}\frac{\partial \omega_\ell}{\partial y_j} - a\beta_j(t)\omega_\ell\right] + a\mu_l\omega_l =$$

$$\sum_{j=1}^{r}\lambda_j\left[a\alpha_j\omega_\ell + i\sqrt{a}\frac{\partial \omega_\ell}{\partial y_j} - a\beta_j(t)\omega_\ell\right]e^{iy_j/\sqrt{a}}$$

$$+aq_\ell(\underline{\beta},\underline{\alpha},\underline{w})\cdot\sum_{j=1}^{r}\mu_j\omega_j e^{-iy_j/\sqrt{a}} - i\sqrt{a}\sum_{j=1}^{r}\mu_j\sum_{k=1}^{r}\frac{\partial q_\ell}{\partial \beta_k}e^{-iy_j/\sqrt{a}}\frac{\partial \omega_j}{\partial y_k} + O(1), \quad (3.11)$$

omitting terms of order $1/a$ in the expansion of q_j. Next divide through (3.11) by a and note that

$$\mu_\ell\omega_\ell - q_\ell\sum_{j=1}^{r}\mu_j\omega_j e^{-iy_j/\sqrt{a}} + \sum_{j=1}^{r}\left[\lambda_j(\alpha_j-\beta_j)\left(1-e^{iy_j/\sqrt{a}}\right) + \frac{i}{\sqrt{a}}\frac{d\beta_j}{dt}y_j\right]\omega_\ell$$

$$+\frac{i}{\sqrt{a}}\sum_{j=1}^{r}\mu_j\sum_{k=1}^{r}\frac{\partial q_\ell}{\partial \beta_k}\frac{\partial \omega_j}{\partial y_k}e^{-iy_j/\sqrt{a}} + \frac{i}{\sqrt{a}}\sum_{j=1}^{r}\lambda_j\frac{\partial \omega_\ell}{\partial y_j}\left(1-e^{iy_j/\sqrt{a}}\right) = O(1/a). \quad (3.12)$$

Expand the exponentials to find

$$\mu_\ell\omega_\ell - q_\ell\sum_{j=1}^{r}\mu_j\omega_j + \frac{i}{\sqrt{a}}\sum_{j=1}^{r}\left[\frac{d\beta_j}{dt} - \lambda_j(\alpha_j-\beta_j)\right]y_j\omega_\ell =$$

$$-\frac{i}{\sqrt{a}}\sum_{j=1}^{r}\mu_j\left[q_\ell\omega_j y_j + \sum_{k=1}^{r}\frac{\partial q_\ell}{\partial \beta_k}\frac{\partial \omega_j}{\partial y_k}\right] + O(1/a). \quad (3.13)$$

If (3.11) is summed on ℓ then, noting that since $\sum_{l=1}^{r}q_l(\underline{\beta},\underline{\alpha}) = 1$, so $\sum_{\ell=1}^{r}\partial q_\ell/\partial\beta_k = 0$, *etc.*, there results

$$\sum_{j=1}^{r}\mu_j\left(1-e^{-iy_j/\sqrt{a}}\right)\omega_j + \sum_{j=1}^{r}\left[\lambda_j(\alpha_j-\beta_j)\left(1-e^{iy_j/\sqrt{a}}\right) + \frac{i}{\sqrt{a}}\frac{d\beta_j}{dt}\cdot y_j\right]\sum_{l=1}^{r}\omega_\ell$$

$$+\frac{i}{\sqrt{a}}\sum_{j=1}^{r}\lambda_j\left(1-e^{iy_j/\sqrt{a}}\right)\sum_{\ell=1}^{r}\frac{\partial \omega_\ell}{\partial y_j} + \frac{1}{a}\sum_{\ell=1}^{r}\frac{\partial \omega_\ell}{\partial t} = 0. \quad (3.14)$$

The fact that this summed equation holds follows directly from the summed Kolmogorov equation; details are omitted. Now expand the exponentials in powers of $1/\sqrt{a}$ to find

$$\frac{i}{\sqrt{a}}\left\{\sum_{j=1}^{r}\mu_j\omega_j y_j+\sum_{j=1}^{r}\left[\frac{d\beta_j}{dt}-\lambda_j\left(\alpha_j-\beta_j\right)\right]y_j\left(\sum_{\ell=1}^{r}\omega_\ell\right)\right\}$$

$$+\frac{1}{a}\left\{\sum_{j=1}^{r}\mu_j\omega_j\frac{y_j^2}{2}+\sum_{j=1}^{r}\lambda_j\left(\alpha_j-\beta_j\right)\frac{y_j^2}{2}\left(\sum_{\ell=1}^{r}\omega_\ell\right)+\sum_{j=1}^{r}\lambda_j y_j\left(\sum_{\ell=1}^{r}\frac{\partial\omega_\ell}{\partial y_j}\right)+\sum_{\ell=1}^{r}\frac{\partial\omega_\ell}{\partial t}\right\}$$

$$=O\left(1/a^{3/2}\right). \tag{3.15}$$

Expansions

Next introduce the asymptotic expansion

$$\omega_\ell\left(\underline{y},t;a\right)=\omega_\ell^{(0)}\left(\underline{y},t\right)+\frac{1}{\sqrt{a}}\omega_\ell^{(1)}\left(\underline{y},t\right)+\frac{1}{a}\omega_\ell^{(2)}\left(\underline{y},t\right)+\ldots \tag{3.16}$$

Insert (3.16) into (3.13) and isolate terms of O(1):

$$\mu_\ell\omega_\ell^{(0)}-q_\ell\sum_{j=1}^{r}\mu_j\omega_j^{(0)}=0,$$

thence, with

$$\Omega^{(0)}\left(\underline{y},t\right)=\sum_{j=1}^{r}\mu_j\omega_j^{(0)}, \tag{3.17}$$

we obtain

$$\omega_\ell^{(0)}=\frac{q_\ell}{\mu_\ell}\Omega^{(0)}\left(\underline{y},t\right). \tag{3.18}$$

In the summed equation (3.15) we have

$$\sum_{j=1}^{r} y_j\left[\mu_j\omega_j^{(0)} + \left(\frac{d\beta_j}{dt} - \lambda_j(\alpha_j - \beta_j)\left(\sum_{\ell=1}^{r}\omega_\ell^{(0)}\right)\right)\right] = 0. \tag{3.19}$$

Put

$$\varphi \equiv \varphi(\underline{\beta},\underline{\alpha}) = \sum_{\ell=1}^{r} q_\ell / \mu_\ell; \tag{3.20}$$

then (3.19) becomes

$$\sum_{j=1}^{r} y_j\left[q_j + \left(\frac{d\beta_j}{dt} - \lambda_j(\alpha_j - \beta_j)\right)\varphi\right]\Omega^{(0)} = 0,$$

which is satisfied if

$$\frac{d\beta_j}{dt} = \lambda_j(\alpha_j - \beta_j) - q_j / \varphi$$

$$= \lambda_j(\alpha_j - \beta_j) - \mu_j\left(\frac{q_j / \mu_j}{\sum_{j=1}^{r} q_j / \mu_j}\right). \tag{3.21}$$

The solution of (3.21) provides the initial approximation to the mean length of the j^{th} queue: $E[N_j(t)] \sim a\beta_j(t)$.

Note that $\chi^{(0)}(\underline{y},t) = \sum_{\ell=1}^{r}\omega_\ell^{(0)} = \varphi\Omega^{(0)}(\underline{y},t)$ provides the leading term in the asymptotic expansion for the ch.f. of $\underline{V}(t)$, the stochastic noise term that perturbs the mean and that $\chi^{(0)}(\underline{0},t) = 1$.

Now return to (3.13) and isolate terms of order $1/\sqrt{a}$ to find that

$$\mu_\ell\omega_\ell^{(1)} - q_\ell\sum_{j=1}^{r}\mu_j\omega_j^{(1)} = -i\sum_{j=1}^{r}\mu_j\left[q_\ell\omega_j^{(0)}y_j + \sum_{k=1}^{r}\frac{\partial q_\ell}{\partial\beta_k}\cdot\frac{\partial\omega_j^{(0)}}{\partial y_k}\right]$$

$$-i\sum_{j=1}^{r}\left[\frac{d\beta_j}{dt} - \lambda_j(\alpha_j - \beta_j)\right]y_j\omega_\ell^{(0)}; \tag{3.22}$$

replace $\omega_j^{(0)}$ by $\left(q_j / \mu_j\right)\Omega^{(0)}$ and utilize $\sum_{j=1}^{r} q_j = 1$ and (3.21) to find

$$\mu_\ell \omega_\ell^{(1)} - q_\ell \sum_{j=1}^{r} \mu_j \omega_j^{(1)} = -i\left[\sum_{k=1}^{r} \frac{\partial q_\ell}{\partial \beta_k} \frac{\partial \Omega^{(0)}}{\partial y_k} + q_\ell \Omega^{(0)} \sum_{j=1}^{r} q_j y_j\right] + i \frac{q_\ell}{\mu_\ell} \frac{\Omega^{(0)}}{\varphi} \sum_{j=1}^{r} q_j y_j. \quad (3.23)$$

In order to make use of the summed equation, (3.15), first put $\Omega^{(1)}\left(\underline{y},t\right) = \sum_{j=1}^{r} \mu_j \omega_j^{(1)}\left(\underline{y},t\right)$ in (3.22), divide through by μ_ℓ and sum to obtain

$$\sum_{\ell=1}^{r} \omega_\ell^{(1)} = \varphi\Omega^{(1)} - i\left[\sum_{k=1}^{r} \frac{\partial \varphi}{\partial \beta_k} \frac{\partial \Omega^{(0)}}{\partial y_k} + \left(\varphi - \frac{R}{\varphi}\right)\Omega^{(0)} \sum_{k=1}^{r} q_k y_k\right] \quad (3.24)$$

in which

$$R = \sum_{\ell=1}^{r} q_\ell / \mu_\ell^2. \quad (3.25)$$

Multiply (3.23) by y_ℓ and sum:

$$\sum_{\ell=1}^{r} y_\ell \mu_\ell \omega_\ell^{(1)} = \sum_{\ell=1}^{r} q_\ell y_\ell \sum_{j=1}^{r} \mu_j \omega_j^{(1)} - i\left[\sum_{\ell=1}^{r}\sum_{k=1}^{r} y_\ell \frac{\partial q_\ell}{\partial \beta_k} \frac{\partial \Omega^{(0)}}{\partial y_k} + \Omega^{(0)}\left(\sum_{j=1}^{r} q_j y_j\right)^2\right]$$
$$+ i \frac{\Omega^{(0)}}{\varphi} \sum_{\ell=1}^{r} \frac{q_\ell y_\ell}{\mu_\ell} \sum_{k=1}^{r} q_j y_j. \quad (3.26)$$

Now utilize (3.26) and (3.24) in the summed equation, (3.15), and put

$$\Omega^{(1)} = \sum_{j=1}^{r} \mu_j \omega_j^{(1)} \quad (3.27)$$

to obtain

$$
\begin{aligned}
& i\Bigg\{\underline{\Omega^{(1)}\sum_{\ell=1}^{r} q_\ell y_\ell} - i\left[\sum_{\ell=1}^{r}\sum_{k=1}^{r} y_\ell \frac{\partial q_\ell}{\partial \beta_k}\frac{\partial \Omega^{(0)}}{\partial y_k} + \underline{\underline{\Omega^{(0)}\left(\sum_{j=1}^{r} q_j y_j\right)^2}} - \frac{\Omega^{(0)}}{\varphi}\sum_{\ell=1}^{r}\frac{q_\ell y_\ell}{\mu_\ell}\sum_{j=1}^{r} q_j y_j\right] \\
& -\sum_{j=1}^{r}\frac{q_j y_j}{\varphi}\left(\underline{\varphi\Omega^{(1)}} - i\left[\sum_{k=1}^{r}\frac{\partial \varphi}{\partial \beta_k}\frac{\partial \Omega^{(0)}}{\partial y_k} + \underline{\underline{\varphi\Omega^{(0)}\sum_{k=1}^{r} q_k y_k}} - \frac{R}{\varphi}\Omega^{(0)}\sum_{k=1}^{r} q_k y_k\right]\right)\Bigg\} \\
& +\left\{\sum_{j=1}^{r}\mu_j\omega_j^{(0)}\frac{y_j^2}{2} + \sum_{j=1}^{r}\lambda_j(\alpha_j-\beta_j)\frac{y_j^2}{2}\left(\sum_{\ell=1}^{r}\omega_l^{(0)}\right) + \sum_{j=1}^{r}\lambda_j y_j\left(\sum_{\ell=1}^{r}\frac{\partial \omega_\ell^{(0)}}{\partial y_j}\right) + \sum_{\ell=1}^{r}\frac{\partial \omega_\ell^{(0)}}{\partial t}\right\} \\
& = 0.
\end{aligned}
\tag{3.28}
$$

The terms designated by $\underline{\quad}$ and by $\underline{\underline{\quad}}$ cancel, leaving an equation involving only the first terms in the expansion (3.16). Further, in view of (3.18), the latter can be reduced to an equation involving only $\Omega^{(0)}(\underline{y},t)$.

If we define

$$
s_j(\underline{\beta},\underline{\alpha}) = q_j(\underline{\beta},\underline{\alpha}) / \varphi(\underline{\beta},\underline{\alpha}), \qquad r(\underline{\beta},\underline{\alpha}) = R(\underline{\beta},\underline{\alpha}) / \varphi(\underline{\beta},\underline{\alpha}) \tag{3.29}
$$

and recall, $\chi^{(0)}(\underline{y},t) = \varphi(\underline{\beta},\underline{\alpha})\Omega^{(0)}(\underline{y},t)$ then (3.28) is expressed as follows:

$$
\begin{aligned}
\frac{\partial \chi^{(0)}}{\partial t} &= -\left(\sum_{j=1}^{r}\lambda_j y_j \frac{\partial \chi^{(0)}}{\partial y_j} + \sum_{j=1}^{r}\sum_{k=1}^{r}\frac{\partial s_j}{\partial \beta_k} y_j \frac{\partial \chi^{(0)}}{\partial y_k}\right) \\
&\left(-\frac{1}{2}\sum_{j=1}^{r}\left[\lambda_j(\alpha_j-\beta_j)+s_j\right]y_j^2 + \sum_{j=1}^{r}\sum_{k=1}^{r}\left[\frac{1}{\mu_j} - r(\underline{\beta},\underline{\alpha})\right]s_j s_k y_j y_k\right)\chi^{(0)}.
\end{aligned}
\tag{3.30}
$$

Equation (3.30) is recognizable as the partial differential equation satisfied by the ch.f. of the Ornstein-Uhlenbeck process. This shows that the scaled noise terms have normal/Gaussian marginal distributions, and permits the derivation of a system of differential equations for the covariance function of the stochastic queue lengths. These are obtained by differentiating (3.30) at $\underline{y} = 0$: if $\rho_{j\ell}(t)$ is the limiting covariance of $V_j(t)$, $V_\ell(t)$, as defined in (3.3), then

$$\frac{d}{dt}\rho_{j\ell}(t) + \left(\lambda_j + \lambda_\ell\right)\rho_{j\ell}(t) + \sum_{k=1}^{r}\left[\frac{\partial s_j}{\partial \beta_k}\cdot \rho_{\ell k}(t) + \frac{\partial s_\ell}{\partial \beta_k}\cdot \rho_{jk}(t)\right] = \left[2r\left(\underline{\beta},\underline{\alpha}\right) - \frac{1}{\mu_j} - \frac{1}{\mu_\ell}\right]s_j s_\ell + \left[\lambda_j\left(\alpha_j - \beta_j\right) + s_j\right]\delta_{j\ell} \tag{3.31}$$

with $\delta_{j\ell} = 1$ if $j = \ell$, $= 0$ if $j \neq \ell$ (Kronecker delta). We have used the fact that $\chi^{(0)}(\underline{0},t) = 1$. A first-order correction term to the mean can also be derived, as can further terms in the asymptotic expansion: if

$$E[N_j(t)] = a\beta_j(t) + \sqrt{a}\,\xi_j(t) + 0(1)$$

then

$$\frac{d\xi_j(t)}{dt} + \lambda_j \xi_j(t) + \sum_{k=1}^{r}\frac{\partial s_j}{\partial \beta_k}\xi_k(t) = 0; \tag{3.32}$$

note that if $\xi_j(0) = 0$, $j = 1, 2, ..., r$, then $\xi_j(t) \equiv 0$.

It seems likely that under the normalization chosen the process $\underline{V}(t)$ actually converges weakly to the Ornstein-Uhlenbeck process, but we do not prove this fact here; see McNeil and Schach (1973) for an early approach to this issue.

We now turn to a numerical assessment of the quality of the approximation for the time-dependent mean and standard deviation of individual queue lengths, and a discussion of the effect of the several service scheduling rules.

4. NUMERICAL RESULTS

The asymptotic results will now be illustrated numerically, and compared to simulation. Two sample systems will be examined.

EXAMPLE 1: CONFIGURATION AND RATES

	Machines				
j:	1	2	3	4	5
K_j:	100	110	120	130	140
λ_j:	0.011	0.012	0.013	0.014	0.015
υ_j:	3.0	3.0	3.0	3.0	3.0

The following abbreviations identify the various service disciplines and computational procedures in the tables:

- FCFS,A/S: First Come, First-Served; Analytical Method/Simulation
- LOLIN,A/S: Longest Line Next; Analytical Method/Simulation
- LAIN,A/S: Least Available Item Next; Analytical Method/Simulation

The simulation has been carried out so as to faithfully represent the service disciplines: FCFS,S literally keeps track of the order of arrival and serves accordingly, and LOLIN,S and LAIN,S likewise perform as stated when new service opportunities occur; they occur deterministically. The analytical procedures utilize the probabilistic schemes described. In all present cases initial conditions are nominally zero; in order to avoid problems with indeterminacy of q_i small values of $\beta_j(0)$ were utilized; weights w_j were taken to be equal. The three times selected to report results essentially cover the interesting range of temporal variation; the system is nearly in statistical equilibrium at the final value. Simulations were replicated 500 times; for details see Pilnick (1989). The typical simulation exercise for a single case, e.g., FCFS,S for all machines required about ten minutes of IBM 3033/4381 time, while the corresponding analytical-numerical solution was carried out in the order of a few seconds.

Table 1 allows comparison of the mean and standard deviation (parentheses) of queue lengths as (a) these measures are computed analytically-numerically, solving equations (3.20) and (3.26), and (b) by point-event simulation. It also invites comparison of the effect of the various disciplines.

Discussion

Figures in Table 1 show that

- For all disciplines employed, and at all times recorded, the analytical-numerical and simulated means and standard deviations agreed well. Of course the analytical-numerical computations were performed in a small fraction of simulation time. Note that the target p-value (tuning parameter) p = 30 provides numbers closer to simulation despite the danger of instability associated with very large p's. Choice of a and of p must be guided by experiment at present.
- The stochastic variability around the time-dependent mean queue length, measured by the standard deviations, is always a small fraction of the mean. The FCFS standard deviations are always somewhat greater than those for the LOLIN or LAIN policies; these latter have a natural tendency to equalize queue lengths, both with respect to means *and* stochastic fluctuations. Studies not reported here, Pilnick (1989), suggest that the scaled marginal distribution of queue length, $V_j(t)$, is close to being normally distributed, as would be implied by the Ornstein-Uhlenbeck nature of the limiting ch.f., (3.30).
- The LAIN policy quickly adapts to equalize the number of un-queued, or available or idle machines in each group. This feature is of special interest when one available machine of each type is needed on a "platform" (e.g., ship, aircraft, *etc.*) in order that it be totally mission-ready. For other models and studies in this direction see Gaver *et al.*, (1991). Note that LAIN is myopic and makes no use of estimates of parameter values λ_j or μ_j, or even the number, r, of

competing machine types. The same general objective should be of interest if the machine types are components of a sequential manufacturing process and it is desired to avoid bottlenecks.

TABLE 1

MACHINE	SERVICE DISCIPLINES		TIMES 100	200	500
1	FCFS,S		40.5(5.3)	52.4(5.3)	57.2(5.1)
	FCFS,A		40.3(5.2)	52.6(5.3)	57.5(5.2)
	LOLIN,S		53.5(3.3)	68.7(3.1)	74.0(3.2)
	LOLIN,A	(p = 10)	50.9(3.7)	65.3(3.5)	70.8(3.4)
		(p = 30)	53.2(3.7)	68.0(3.4)	73.5(3.2)
	LAIN,S		NA	NA	NA
	LAIN,A	(p = 30)	33.1(3.7)	47.9(3.4)	53.5(3.2)
2	FCFS,S		47.4(5.5)	60.6(5.7)	65.5(5.6)
	FCFS,A		47.0(5.6)	60.5(5.6)	65.6(5.4)
	LOLIN,S		54.3(3.4)	69.2(3.1)	74.5(3.3)
	LOLIN,A	(p = 10)	53.0(3.9)	67.6(3.7)	73.1(3.6)
		(p = 30)	54.1(3.6)	68.6(3.4)	74.5(3.3)
	LAIN,S		NA	NA	NA
	LAIN,A	(p=30)	43.4(3.7)	58.1(3.4)	63.6(3.2)
3	FCFS,S		53.8(6.3)	68.5(5.8)	73.7(6.0)
	FCFS,A		54.0(5.9)	68.6(5.8)	73.8(5.7)
	LOLIN,S		54.8(3.5)	69.6(3.1)	74.9(3.1)
	LOLIN,A	(p = 10)	54.7(4.1)	69.6(3.9)	75.2(3.8)
		(p = 30)	53.7(3.7)	69.7(3.6)	75.3(3.5)
	LAIN,S		NA	NA	NA
	LAIN,A	(p = 30)	53.7(3.7)	68.2(3.4)	73.7(3.2)
4	FCFS,S		61.2(6.6)	76.9(6.1)	81.6(5.9)
	FCFS,A		61.2(6.2)	76.9(6.0)	82.3(5.8)
	LOLIN,S		55.4(3.6)	70.1(3.2)	75.3(3.3)
	LOLIN,A	(p = 10)	56.2(4.2)	71.3(4.1)	77.0(4.0)
		(p = 30)	55.2(3.7)	70.2(3.6)	75.9(3.5)
	LAIN,S		NA	NA	NA
	LAIN,A	(p = 30)	63.9(3.7)	78.4(3.4)	83.8(3.2)
5	FCFS,S		68.4(6.8)	84.4(6.1)	90.3(6.0)
	FCFS,A		68.8(6.5)	85.3(6.2)	90.8(6.0)
	LOLIN,S		55.9(3.6)	70.6(3.2)	75.8(3.3)
	LOLIN,A	(p = 10)	57.5(4.4)	72.9(4.3)	78.7(4.2)
		(p = 30)	55.7(3.8)	70.8(3.7)	76.5(3.5)
	LAIN,S		NA	NA	NA
	LAIN,A	(p = 30)	74.2(3.7)	88.5(3.4)	93.9(3.2)

EXAMPLE 2: CONFIGURATION AND RATES
(Failure and Repair Rates Differ)

	Machines				
j:	1	2	3	4	5
K_j:	100	110	120	130	140
λ_j:	0.015	0.020	0.025	0.030	0.035
υ_j:	1.0	1.1	1.2	1.3	1.4

Note that in this example the repair rates differ, whereas in the previous example they were the same. In Table 2 there appear the analytical and simulated means and standard deviation (parentheses) of the individual machine availabilities when the FCFS and LAIN service policies are in effect.

TABLE 2. MACHINES AVAILABLE

MACHINES	SERVICE DISCIPLINES	TIMES		
		50	100	200
1	FCFS,S	51.5(5.0)	28.9(4.6)	13.9(3.3)
	FCFS,A	52.1(5.0)	30.3(4.6)	16.1(3.7)
	LAIN,S	51.3(4.7)	24.5(3.9)	11.7(1.6)
	LAIN,A (p = 10)	52.1(4.8)	25.2(3.9)	11.5(1.7)
2	FCFS,S	46.1(5.0)	22.8(4.5)	11.6(3.3)
	FCFS,A	46.5(5.2)	23.6(4.3)	12.5(3.3)
	LAIN,S	43.9(4.3)	20.0(2.2)	11.3(1.6)
	LAIN,A (p = 10)	44.9(4.4)	20.2(2.6)	11.1(1.7)
3	FCFS,S	41.0(5.2)	18.6(4.2)	10.7(3.3)
	FCFS,A	41.4(5.3)	18.9(4.0)	10.5(3.1)
	LAIN,S	40.1(3.4)	19.3(2.5)	11.0(1.7)
	LAIN,A (p = 10)	40.2(3.9)	19.2(2.5)	10.8(1.7)
4	FCFS,S	51.5(5.0)	28.9(4.6)	13.9(3.3)
	FCFS,A	52.1(5.0)	30.3(4.6)	16.1(3.7)
	LAIN,S	51.3(4.7)	24.5(3.9)	11.7(1.6)
	LAIN,A (p = 10)	52.1(4.8)	25.2(3.9)	11.5(1.7)
5	FCFS,S	51.5(5.0)	28.9(4.6)	13.9(3.3)
	FCFS,A	52.1(5.0)	30.3(4.6)	16.1(3.7)
	LAIN,S	51.3(4.7)	24.5(3.9)	11.7(1.6)
	LAIN,A (p = 10)	52.1(4.8)	25.2(3.9)	11.5(1.7)

Discussion

Table 2 indicates that the analytical-numerical and simulation-generated means and standard deviations again agree reasonably well, even though individual type failure rates and repair rates differ. The major discrepancies seem to appear in FCFS discipline comparisons. The qualitative effects noted earlier are once again evident: generally small standard deviation to mean ratio (coefficient of variation); larger standard deviation to mean ratio (coefficient of variation) and larger standard deviation for FCFS than for LAIN, and a pronounced (an anticipated) tendency for LAIN to equalize the number of machines available as time advances.

Conclusions

Our conclusion is that asymptotic expressions based on the scaling (3.3) provide quite adequate approximations to the time-dependent or transient means and variances for the particular multitype repairman problems studied. These problems include attention to a variety of dynamic priority or service-adaptive policies via the device of tailored probabilistic selection of new service incumbents. Not surprisingly, convergence to the marginal distribution of an Ornstein-Uhlenbeck process occurs; actual weak convergence has not yet been demonstrated, but is a likely bonus.

REFERENCES

Ludwig Arnold (1974). *Stochastic Differential Equations, Theory and Applications.* Wiley-Interscience. John Wiley and Sons, N.Y.

D. R. Cox and W. L. Smith (1961). *Queues.* Methuen Monograph. London.

W. Feller (1967). *An Introduction to Probability Theory and its Applications.* John Wiley and Sons, N.Y. (Third Edition).

D. P. Gaver, K. Isaacson, S. E. Pilnick and R. Silveira (1991). Assessing and improving many-platform availability during intensive demand periods. (paper in preparation).

D. P. Gaver and P. A. Jacobs (1986). Processor-shared time-sharing models in heavy traffic. *SIAM J. Comput.,* **15**, pp. 1085-1100.

D. P. Gaver and J. A. Morrison (1991). Heavy-traffic analysis of multi-type queueing under probabilistically load-preferential service order. *SIAM J. Appl. Math.* (to appear).

M. P. Jaiswal, C. L. Sharma, Karmeshu (1990). A queueing model with hysteresis effect: an application to a machine interference problem. Unpublished paper.

D. R. McNeil and S. Schach (1973). Central limit analogues for Markov population processes, *J. of the Royal Stat. Soc.* (B), No. 1, pp. 1-23.

J. A. Morrison (1987). Conditioned response-time distribtuion for a large closed processor-sharing system in very heavy usage. *SIAM J. Appl. Math.,* **47**, pp. 1117-1129.

S. E. Pilnick (1989). *Combat Logistics Problems.* Ph.D. Dissertation, Dept. of Operations Research, Naval Postgraduate School, Monterey, CA.

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